Entry Task: Differentiate

1. $F(x) = \frac{7}{x^{10}} - 5\sqrt{x^3} + 4\ln(x) = 7x^{-10} - 5x^{3/2} + 4\ln(x)$
2. $G(x) = e^{6x} + 5\tan(x) + \pi$
3. $H(x) = 2\tan^{-1}(x) - 3 + e$
4. $J(x) = x^3\cos(4x) + \ln(2)$

$\boxed{1}$ $F'(x) = 7(-10x^{-11}) - 5\left(\frac{3}{2}x^{1/2}\right) + 4\left(\frac{1}{x}\right) = -70x^{-11} - \frac{15}{2}x^{1/2} + \frac{4}{x}$

$\boxed{2}$ $G'(x) = 6e^{6x} + 5\sec^2(x) + 0 \quad = 6e^{6x} + 5\sec^2(x)$

$\boxed{3}$ $H'(x) = 2\,\frac{1}{1+x^2} - 0 + 0 \quad = \frac{2}{1+x^2}$

$\boxed{4}$ $J'(x) = 3x^2\cos(4x) - 4x^3\sin(4x) + 0$

4.9 Antiderivatives

Goal: Before we jump into defining integrals (ch. 5), we need to remember some derivatives (in reverse).

Def'n: If g(x) = f'(x), then we say

g(x) = "<u>the derivative</u> of f(x)", and

f(x) = "<u>an antiderivative</u> of g(x)"

Example:

Give an antiderivative of $g(x) = x^2$.

OOPS

POWER UP ONE

GUESS:

- $G(x) = x^3$ CHECK: $G'(x) = 3x^2$

DOESN'T WORK!

Now try

$G(x) = \frac{1}{3}x^3$

CHECK: $G'(x) = \frac{1}{3}3x^2 = x^2$ ✓✓

$G(x) = \frac{1}{3}x^3$ IS ONE ANTIDERIVATIVE

<u>BUT</u>, THERE ARE OTHERS

$\frac{1}{3}x^3 + 14$, $\frac{1}{3}x^3 + 17$, $\frac{1}{3}x^3 - 105$, ...

GENERAL ANTIDERIVATIVE IS $G(x) = \frac{1}{3}x^3 + C$ where C is a constant

4.9: LIST OF GENERAL ANTIDERIVATIVES

FUNCTION	ANTIDERIVATIVE
$f(x) = x^n \ (n \neq -1)$	$F(x) = \frac{1}{n+1}x^{n+1} + C$
$f(x) = x^{-1} = \frac{1}{x}$	$F(x) = \ln\lvert x\rvert + C$
$f(x) = e^x$	$F(x) = e^x + C$
$f(x) = \cos(x)$	$F(x) = \sin(x) + C$
$f(x) = \sec^2(x)$	$F(x) = \tan(x) + C$
$f(x) = \sec(x)\tan(x)$	$F(x) = \sec(x) + C$
$f(x) = \sin(x)$	$F(x) = -\cos(x) + C$
$f(x) = \csc^2(x)$	$F(x) = -\cot(x) + C$
$f(x) = \csc(x)\cot(x)$	$F(x) = -\csc(x) + C$
$f(x) = \frac{1}{1+x^2}$	$F(x) = \tan^{-1}(x) + C$

Examples (you do):

Find the general antiderivative of

1. $f(x) = x^6$
2. $g(x) = \cos(x) + \frac{1}{x} + e^x + \frac{1}{1+x^2}$
3. $h(x) = \frac{5}{\sqrt{x}} + \sqrt[3]{x^2} = 5x^{-1/2} + x^{3/2}$
4. $r(x) = \frac{x-3x^2}{x^3} = \frac{x}{x^3} - \frac{3x^2}{x^3} = \frac{1}{x^2} - \frac{3}{x} = x^{-2} - 3 \cdot \frac{1}{x}$

$\boxed{1}$ $F(x) = \frac{1}{7}x^7 + C$

$\boxed{2}$ $G(x) = \sin(x) + \ln(x) + e^x + \tan^{-1}(x) + C$

$\boxed{3}$ $H(x) = 5\left(\frac{1}{1/2}x^{1/2}\right) + \left(\frac{1}{5/2}x^{5/2}\right) = 10\sqrt{x} + \frac{2}{5}x^{5/2} + C$

$\boxed{4}$ $R(x) = \left(\frac{1}{-1}x^{-1}\right) - 3\ln(x) + C = -\frac{1}{x} - 3\ln(x) + C$

Initial Conditions

There is no way to know what "C" is unless we are given additional information about the antiderivative. Such information is called an **initial condition**.

Example: $f'(x) = e^x + 4x$ and $f(0) = 5$

Find $f(x)$.

STEP 1

$$F(x) = e^x + 4\cdot\tfrac{1}{2}x^2 + C$$

$$F(x) = e^x + 2x^2 + C$$

STEP 2

$$f(0) = 5$$

$$\Rightarrow e^{(0)} + 2(0)^2 + C = 5$$

$$\Rightarrow 1 + 0 + C = 5$$

$$\Rightarrow \boxed{C = 4}$$

$$\boxed{f(x) = e^x + 2x^2 + 4}$$

CHECK!!

Example: $f''(x) = 15\sqrt{x}$, and $f(1) = 0, f(4) = 1$

Find $f(x)$.

STEP 1

$$f''(x) = 15x^{1/2}$$

$$\Rightarrow f'(x) = 15\cdot\tfrac{2}{3}x^{3/2} + C$$

$$f'(x) = 10x^{3/2} + C$$

unknown constant

STEP 2

$$\boxed{f(x) = 10\left(\tfrac{2}{5}x^{5/2}\right) + Cx + D}$$

$$\boxed{f(x) = 4x^{5/2} + Cx + D}$$

STEP 3

$$f(1) = 0 \Rightarrow 4(1)^{5/2} + C(1) + D = 0$$

$$\Rightarrow 4 + C + D = 0 \Rightarrow D = -4 - C$$

$$f(4) = 1 \Rightarrow 4(4)^{5/2} + C(4) + D = 1$$

$$128 + 4C + D = 1$$

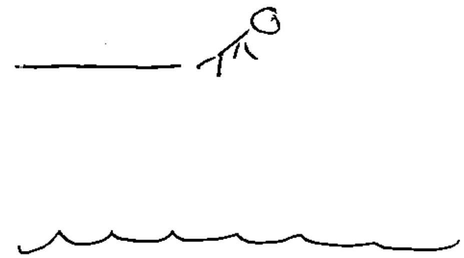
$$128 + 4C + (-4 - C) = 1$$

$$3C = -123$$

$$\boxed{\begin{array}{l} C = -\tfrac{123}{3} = -41 \\ D = 37 \end{array}}$$

Motivational: You know the acceleration or velocity function for some object. What is the original function for the position of the object?

Example:
Ron *steps off* the 10 meter high dive at his local pool. Find a formula for his height above the water.
(Assume his acceleration is a constant 9.8 m/s^2 downward)



$h(t)$ = height at time t seconds

$h(0) = 10$

$v(0) = 0 \leftarrow$ initial velocity "steps off"

$a(t) = -9.8$

$\Rightarrow h''(t) = -9.8$

$\Rightarrow h'(t) = -9.8t + C$

$\Rightarrow h(t) = -4.9t^2 + Ct + D$

$v(0) = 0 \Rightarrow -9.8(0) + C = 0 \Rightarrow C = 0$

$h(0) = 10 \Rightarrow -4.9(0)^2 + D = 10 \Rightarrow D = 10$

$$\boxed{h(t) = -4.9t^2 + 10}$$